Calculus I
Chapter 1 and 2 Test Review Key

1. Evaluate the expression $\sec(\arctan\frac{2}{3})$ without using a calculator.

First handle inside of parenthesis:

$$P = \arctan\frac{2}{3}$$
$$\tan P = \tan\left(\arctan\frac{2}{3}\right)$$
$$\tan P = \frac{2}{3}$$
$$Quadrant \quad I$$
$$and \quad \tan = \frac{y}{x}$$
$$so \quad y = 2, x = 3$$
$$find \quad r$$
$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{3^2 + 2^2}$$
$$r = \sqrt{9+4}$$
$$r = \sqrt{13}$$

Second handle outer trig function

$$\sec = \frac{r}{x}$$
$$\sec = \frac{\sqrt{13}}{3}$$

2. Solve: $3^{x-5} = 27$

$$3^{x-5} = 27$$
$$3^{x-5} = 3^3$$
$$x - 5 = 3$$
$$x = 3 + 5$$
$$x = 8$$

3. Complete the table and use the result to estimate the limit.

$$\lim_{x \to 3} \frac{x-3}{x^2 - 16x + 39}$$

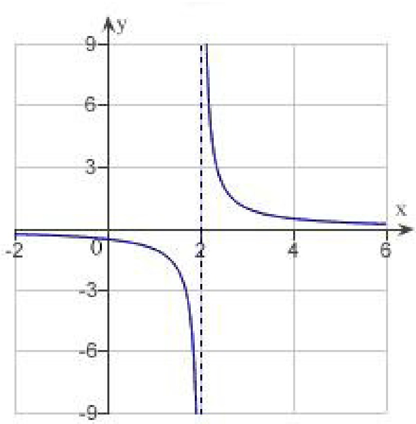
x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$	-0.099	-0.0999	-0.1	-0.1	-0.1001	-0.101

so

$answer = -0.1$

4. Determine the following limit. (Hint: Use the graph to calculate the limit.)

$$\lim_{x \to 2} \frac{1}{x-2}$$



answer = Does Not Exist

5. Find the limit by algebraic evaluation.

$$\lim_{x \to 6} \frac{x}{x^2+8}$$

$$\lim_{x \to 6} \frac{x}{x^2+8}$$
$$= \frac{6}{6^2+8}$$
$$= \frac{6}{36+8}$$
$$= \frac{6}{44}$$
$$= \frac{3}{22}$$

6. Find the limit.

$$\lim_{x \to 5} \cos\left(\frac{\pi x}{6}\right)$$

$$\lim_{x \to 5} \cos\left(\frac{\pi x}{6}\right)$$
$$= \cos\left(\frac{5\pi}{6}\right)$$
$$= \frac{-\sqrt{3}}{2}$$

7. Find the limit (if it exists) using algebraic techniques.

$$\lim_{x \to -8} \frac{x+8}{x^2-64}$$

$$\lim_{x\to -8}\frac{x+8}{x^2-64}$$
$$=\lim_{x\to -8}\frac{x+8}{(x+8)(x-8)}$$
$$=\lim_{x\to -8}\frac{1}{x-8}$$
$$=\frac{1}{-8-8}$$
$$=\frac{1}{-16}$$

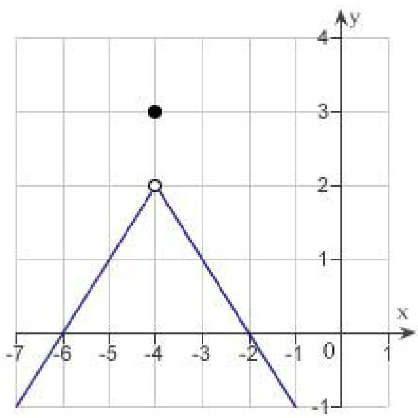
8. Find the limit (if it exists) using the conjugate method.

$$\lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5}$$

$$\lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}$$
$$=\lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}\cdot\frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$$
$$=\lim_{x\to 5}\frac{x+4-9}{(x-5)\left(\sqrt{x+4}+3\right)}$$
$$=\lim_{x\to 5}\frac{x-5}{(x-5)\left(\sqrt{x+4}+3\right)}$$
$$=\lim_{x\to 5}\frac{1}{\sqrt{x+4}+3}$$
$$=\frac{1}{\sqrt{5+4}+3}$$
$$=\frac{1}{\sqrt{9}+3}$$
$$=\frac{1}{3+3}$$
$$=\frac{1}{6}$$

9. Use the graph as shown to determine the following limits, and discuss the continuity of the function at $x = -4$.

(i) $\lim_{x \to -4^+} f(x)$ (ii) $\lim_{x \to -4^-} f(x)$ (iii) $\lim_{x \to -4} f(x)$

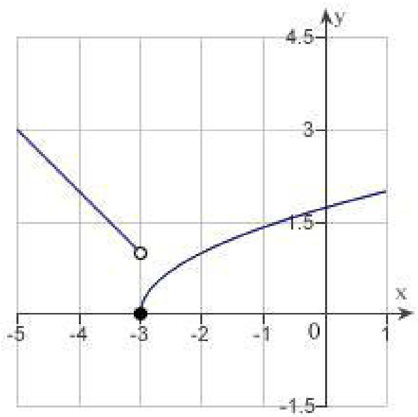


(i) 2
(ii) 2
(iii) 2

discontinuous at x = -4

10. Use the graph to determine the following limits, and discuss the continuity of the function at $x = -3$.

(i) $\lim_{x \to -3^+} f(x)$ (ii) $\lim_{x \to -3^-} f(x)$ (iii) $\lim_{x \to -3} f(x)$



(i) 0
(ii) 1.3
(iii) does not exist

Discontinuous at x = -3

11. Find the limit (if it exists).

$$\lim_{x \to 11^+} \frac{11 - x}{x^2 - 121}$$

$$\lim_{x \to 11^+} \frac{11 - x}{x^2 - 121}$$
$$= \lim_{x \to 11^+} \frac{-x + 11}{(x+11)(x-11)}$$
$$= \lim_{x \to 11^+} \frac{-1(x-11)}{(x+11)(x-11)}$$
$$= \lim_{x \to 11^+} \frac{-1}{x+11}$$
$$= \frac{-1}{11+11}$$
$$= \frac{-1}{22}$$

12. Discuss the continuity of the function $f(x) = \dfrac{x^2 - 4}{x - 2}$.

$$x - 2 = 0$$
$$x = 2$$
$$so$$
$$continuous: (-\infty, 2) \cup (2, \infty)$$

13. Find the x-values (if any) at which the function $f(x) = \dfrac{x + 2}{x^2 + 6x + 8}$ is not continuous. Which of the discontinuities are removable?

$$f(x) = \frac{x+2}{x^2+6x+8}$$
$$f(x) = \frac{x+2}{(x+2)(x+4)}$$
$$so:$$
$$x+2=0 \quad x+4=0$$
$$x=-2 \quad\quad x=-4$$

so discontinuous at -2, -4

$$f(x) = \frac{x+2}{(x+2)(x+4)}$$
$$f(x) = \frac{1}{x+4}$$

since x + 2 cancelled (was removed), x = -2 is a removable discontinuity

14. Find the limit.

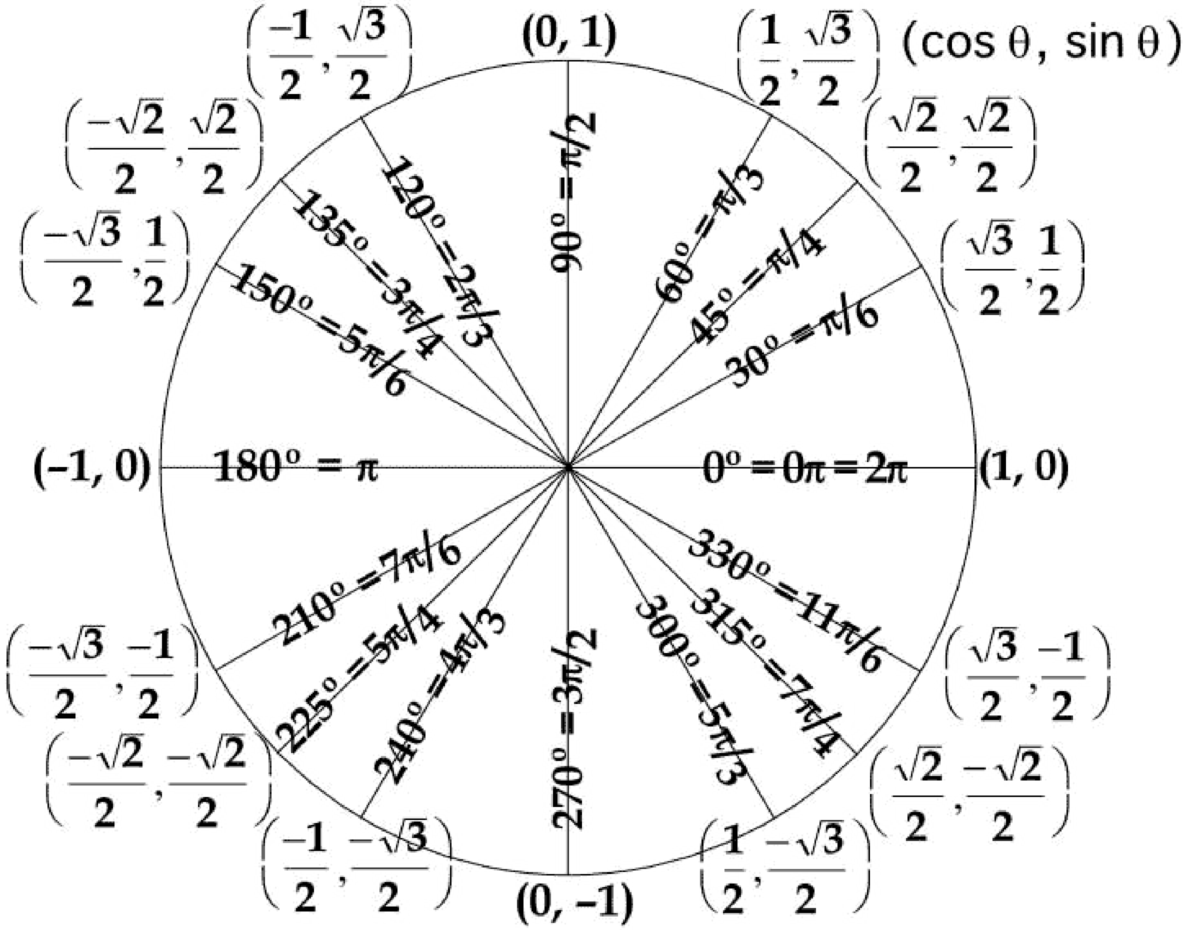
$$\lim_{x \to 14^+} \frac{x-3}{x-14}$$

$answer = \infty$

15. Find the limit.

$$\lim_{x \to 0^-} \left(x^2 - \frac{1}{x}\right)$$

$answer = \infty$


(cos θ, sin θ)
(0, 1)
(1, 0)
(−1, 0)
(0, −1)
90° = π/2
60° = π/3
45° = π/4
30° = π/6
0° = 0π = 2π
120° = 2π/3
135° = 3π/4
150° = 5π/6
180° = π
210° = 7π/6
225° = 5π/4
240° = 4π/3
270° = 3π/2
300° = 5π/3
315° = 7π/4
330° = 11π/6
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
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$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$