6. $\int_1^4 \frac{2}{5x+3}\,dx$

$u = 5x+3 \quad du = 5\,dx$

$= 2 \cdot \frac{1}{5} \int_1^4 \frac{5}{5x+3}\,dx$

$= \frac{2}{5} \int_{x=1}^{x=4} \frac{1}{u}\,du$

$= \frac{2}{5} \left[\ln|u| \right]_{x=1}^{x=4}$

$= \frac{2}{5} \left[\ln|5x+3| \right]_1^4$

$= \frac{2}{5} \left[\ln|5\cdot 4+3| - \ln|5\cdot 1+3| \right]$

$= \frac{2}{5} \left[\ln 23 - \ln 8 \right]$

$= \boxed{\frac{2}{5} \ln \frac{23}{8}}$

7. $\int_0^1 \frac{5}{\sqrt{8x+1}}\,dx$

$u = 8x+1 \quad du = 8\,dx$

$= 5 \cdot \frac{1}{8} \int_0^1 \frac{8}{\sqrt{8x+1}}\,dx$

$= \frac{5}{8} \int_{x=0}^{x=1} \frac{1}{\sqrt{u}}\,du$

$= \frac{5}{8} \int_{x=0}^{x=1} \frac{1}{u^{1/2}}\,du$

$= \frac{5}{8} \int_{x=0}^{x=1} u^{-1/2}\,du$

$= \frac{5}{8} \cdot \left[\frac{u^{-1/2+1}}{-\frac{1}{2}+1} \right]_{x=0}^{x=1}$

$= \frac{5}{8} \cdot \left[\frac{u^{1/2}}{\frac{1}{2}} \right]_{x=0}^{x=1}$

$= \frac{5}{8} \cdot \frac{2}{1} \left[u^{1/2} \right]_{x=0}^{x=1}$

$= \frac{5}{4} \left[\sqrt{u} \right]_{x=0}^{x=1}$

$= \frac{5}{4} \left[\sqrt{8x+1} \right]_0^1$

$= \frac{5}{4} \left[\sqrt{8\cdot 1+1} - \sqrt{8\cdot 0+1} \right]$

$= \frac{5}{4} \left[\sqrt{9} - \sqrt{1} \right]$

$= \frac{5}{4} \left[3 - 1 \right]$

$= \frac{10}{4}$

$= \boxed{\frac{5}{2}}$