$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$$

$$\frac{d}{dx}\left[\ln u\right] = \frac{1}{u}\cdot u'$$

**1.** $f(x) = \ln(x^2 - 2)$

$$f'(x) = \frac{1}{x^2 - 2}\cdot \frac{d}{dx}(x^2 - 2)$$

$$= \boxed{\frac{2x}{x^2 - 2}}$$

**2.** $f(x) = \ln(3x-1)^{\frac{5}{2}}$

$$= \frac{5}{2}\ln(3x-1)$$

$$f'(x) = \frac{5}{2}\cdot \frac{1}{3x-1}\cdot \frac{d}{dx}(3x-1)$$

$$= \boxed{\frac{15}{2(3x-1)}}$$

**3.**

$$f(x) = \ln\left[4x(5x-1)^7\right]$$

$$= \ln 4 + \ln x + \ln(5x-1)^7$$

$$= \ln 4 + \ln x + 7\ln(5x-1)$$

$$f'(x) = \frac{1}{x} + 7\cdot \frac{1}{5x-1}\cdot \frac{d}{dx}(5x-1)$$

$$= \frac{1}{x} + \frac{35}{5x-1}$$

$$= \frac{1(5x-1)}{x(5x-1)} + \frac{35x}{x(5x-1)}$$

$$= \boxed{\frac{40x-1}{x(5x-1)}}$$

**4.** $f(x) = \ln \sqrt[5]{\frac{5x-2}{3x+5}}$

$$= \ln\left(\frac{5x-2}{3x+5}\right)^{\frac{1}{5}}$$

$$= \frac{1}{5}\ln\left(\frac{5x-2}{3x+5}\right)$$

$$= \frac{1}{5}\ln(5x-2) - \frac{1}{5}\ln(3x+5)$$

$$f'(x) = \frac{1}{5}\left[\frac{1}{5x-2}\cdot \frac{d}{dx}(5x-2) - \frac{1}{3x+5}\cdot \frac{d}{dx}(3x+5)\right]$$

$$= \frac{1}{5}\left[\frac{5}{5x-2} - \frac{3}{3x+5}\right]$$

$$= \frac{1}{5}\left[\frac{5(3x+5)}{(5x-2)(3x+5)} - \frac{3(5x-2)}{(5x-2)(3x+5)}\right]$$

$$= \frac{1}{5}\left[\frac{15x+25}{(5x-2)(3x+5)} - \frac{15x-6}{(5x-2)(3x+5)}\right]$$

$$= \frac{1}{5}\left[\frac{15x+25-15x+6}{(5x-2)(3x+5)}\right]$$

$$= \boxed{\frac{31}{5(5x-2)(3x+5)}}$$