5. $f(x) = x^3 - 3x \quad \overset{x}{(1,-2)}$

FINDING SLOPE OF TANGENT LINE AT A POINT USING LIMIT DEF.

① FIND $f'(x)$

ⓐ IDENTIFY $f(x)$

$$f(x) = x^3 - 3x$$

ⓑ FIND $f(x+h)$

$$f(x+h) = (x+h)^3 - 3(x+h)$$
$$= (x+h)(x+h)(x+h) - 3x - 3h$$
$$= (x+h)(x^2 + hx + hx + h^2) - 3x - 3h$$
$$= (x+h)(x^2 + 2hx + h^2) - 3x - 3h$$
$$= x^3 + 2hx^2 + xh^2 + hx^2 + 2h^2x + h^3 - 3x - 3h$$
$$= x^3 + 3hx^2 + 3h^2x + h^3 - 3x - 3h$$

ⓒ $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 - 3x - 3h - (x^3 - 3x)}{h}$$
$$= \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 - 3x - 3h - x^3 + 3x}{h}$$
$$= \lim_{h \to 0} \frac{3hx^2 + 3h^2x + h^3 - 3h}{h}$$
$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3hx + h^2 - 3)}{\cancel{h}}$$
$$= \lim_{h \to 0} 3x^2 + 3hx + h^2 - 3$$
$$f'(x) = 3x^2 + 3(0)x + 0^2 - 3$$
$$f'(x) = 3x^2 - 3$$

② CHANGE $f'(x)$ TO m AND PLUG IN THE x PART OF OUR POINT GIVEN AND SIMPLIFY

$$m = 3(1)^2 - 3$$
$$= 3 - 3$$
$$= 0$$