6. GAUSSIAN ELIMINATION WITH BACK SUBSTITUTION

GOAL $\begin{bmatrix} 1 & \sim & \sim & \sim \\ 0 & 1 & \sim & \sim \\ 0 & 0 & 1 & \sim \end{bmatrix}$

$$\begin{aligned} x - 5z &= -4 \\ 3x + 4y - 2z &= 9 \\ 2x - 4y + 3z &= -3 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & -5 & -4 \\ 3 & 4 & -2 & 9 \\ 2 & -4 & 3 & -3 \end{bmatrix} \xrightarrow[= \text{NEW } R_2]{-3R_1 + R_2} \begin{bmatrix} 1 & 0 & -5 & -4 \\ 0 & 4 & 13 & 21 \\ 2 & -4 & 3 & -3 \end{bmatrix} \xrightarrow[= \text{NEW } R_3]{-2R_1 + R_3} \begin{bmatrix} 1 & 0 & -5 & -4 \\ 0 & 4 & 13 & 21 \\ 0 & -4 & 13 & 5 \end{bmatrix}$$

$$\xrightarrow{R_2/4 = \text{NEW } R_2} \begin{bmatrix} 1 & 0 & -5 & -4 \\ 0 & 1 & \frac{13}{4} & \frac{21}{4} \\ 0 & -4 & 13 & 5 \end{bmatrix} \xrightarrow[= \text{NEW } R_3]{4R_2 + R_3} \begin{bmatrix} 1 & 0 & -5 & -4 \\ 0 & 1 & \frac{13}{4} & \frac{21}{4} \\ 0 & 0 & 26 & 26 \end{bmatrix} \xrightarrow[= \text{NEW } R_3]{R_3 \div 26} \begin{bmatrix} 1 & 0 & -5 & -4 \\ 0 & 1 & \frac{13}{4} & \frac{21}{4} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

ROW ECHELON FORM

$$\begin{bmatrix} x & y & z & = \\ 1 & 0 & -5 & -4 \\ 0 & 1 & \frac{13}{4} & \frac{21}{4} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} 1x + 0y - 5z &= -4 \\ 0x + 1y + \tfrac{13}{4}z &= \tfrac{21}{4} \\ 0x + 0y + 1z &= 1 \end{aligned}$$

$$\begin{aligned} x - 5z &= -4 \\ y + \tfrac{13}{4}z &= \tfrac{21}{4} \\ z &= 1 \end{aligned}$$

$z = 1$

PLUG INTO EQN ABOVE IT

$$y + \tfrac{13}{4}z = \tfrac{21}{4}$$
$$y + \tfrac{13}{4}(1) = \tfrac{21}{4}$$
$$y = \tfrac{21}{4} - \tfrac{13}{4}$$

$y = \frac{8}{4}$

$y = 2$

NOW PLUG INTO EQN. ABOVE THAT ONE

$$x - 5z = -4$$

$$x - 5(1) = -4$$
$$x - 5 = -4$$
$$x = -4 + 5$$
$$x = 1$$

$(1, 2, 1)$