

$$\begin{aligned}
 7. \quad w + x - 2y + 3z &= 16 \\
 3w - x - y + 5z &= 18 \\
 w + 2x - 3y - 4z &= -9 \\
 w - 5x - y - 2z &= -12
 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & -2 & 3 & 16 \\ 3 & -1 & -1 & 5 & 18 \\ 1 & 2 & -3 & -4 & -9 \\ 1 & -5 & -1 & -2 & -12 \end{bmatrix} \rightarrow \begin{array}{l} -3R_1 + R_2 = \text{NEW } R_2 \\ -1R_1 + R_3 = \text{NEW } R_3 \\ -1R_1 + R_4 = \text{NEW } R_4 \end{array} \begin{bmatrix} 1 & 1 & -2 & 3 & 16 \\ 0 & -4 & 5 & -4 & -30 \\ 0 & 1 & -1 & -7 & -25 \\ 0 & -6 & 1 & -5 & -28 \end{bmatrix}$$

SWAP R_2 AND R_3

$$\begin{bmatrix} 1 & 1 & -2 & 3 & 16 \\ 0 & 1 & -1 & -7 & -25 \\ 0 & -4 & 5 & -4 & -30 \\ 0 & -6 & 1 & -5 & -28 \end{bmatrix} \rightarrow \begin{array}{l} 4R_2 + R_3 = \text{NEW } R_3 \\ 6R_2 + R_4 = \text{NEW } R_4 \end{array} \begin{bmatrix} 1 & 1 & -2 & 3 & 16 \\ 0 & 1 & -1 & -7 & -25 \\ 0 & 0 & 1 & -32 & -130 \\ 0 & 0 & -5 & -47 & -178 \end{bmatrix}$$

$\frac{-30}{-160}$ $\frac{130}{-178}$

$$\begin{bmatrix} 1 & 1 & -2 & 3 & 16 \\ 0 & 1 & -1 & -7 & -25 \\ 0 & 0 & 1 & -32 & -130 \\ 0 & 0 & 0 & -207 & -828 \end{bmatrix} \rightarrow \begin{array}{l} 5R_3 + R_4 = \text{NEW } R_4 \\ R_4 \div -207 = \text{NEW } R_4 \end{array} \begin{array}{l} \text{REF} \\ \begin{bmatrix} 1 & 1 & -2 & 3 & 16 \\ 0 & 1 & -1 & -7 & -25 \\ 0 & 0 & 1 & -32 & -130 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \end{array}$$

$\frac{32}{13}$

$$\begin{array}{l} -3R_4 + R_1 = \text{NEW } R_1 \\ 7R_4 + R_2 = \text{NEW } R_2 \\ 32R_4 + R_3 = \text{NEW } R_3 \end{array} \begin{bmatrix} 1 & 1 & -2 & 0 & 4 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \rightarrow \begin{array}{l} 2R_3 + R_1 = \text{NEW } R_1 \\ R_3 + R_2 = \text{NEW } R_2 \end{array} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$$-R_2 + R_1 = \text{NEW } R_1 \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \begin{array}{l} w \\ x \\ y \\ z \end{array}$$

$$(-1, 1, -2, 4)$$

$$\begin{bmatrix} 1 & 1 & -2 & 3 & 16 \\ 3 & -1 & -1 & 5 & 18 \\ 1 & 2 & -3 & -4 & -9 \\ 1 & -5 & -1 & -2 & -12 \end{bmatrix} \quad 4 \times 5$$