5. $x^2 + 16xy + y^2 - 3 = 0$

① $A = 1 \quad B = 16 \quad C = 1$

$$\cot(2\theta) = \frac{A-C}{B}$$

$$\cot(2\theta) = \frac{1-1}{16}$$

$$\cot(2\theta) = 0$$

WHERE IS COT $= 0$

$$\cot 90^\circ = 0$$

SO $2\theta = 90^\circ$

$\theta = 45^\circ$

②

$x = x'\cos\theta - y'\sin\theta \qquad y = x'\sin\theta + y'\cos\theta$

$x = x'\cos 45^\circ - y'\sin 45^\circ \qquad y = x'\sin 45^\circ + y'\cos 45^\circ$

$x = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y' \qquad y = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$

$x = \frac{\sqrt{2}}{2}(x' - y') \qquad y = \frac{\sqrt{2}}{2}(x' + y')$

LET $P = x'$ AND $Q = y'$

$x = \frac{\sqrt{2}}{2}(P - Q) \qquad y = \frac{\sqrt{2}}{2}(P + Q)$

③ $x^2 + 16xy + y^2 - 3 = 0$

$$\left[\frac{\sqrt{2}}{2}(P-Q)\right]^2 + 16\left[\frac{\sqrt{2}}{2}(P-Q)\frac{\sqrt{2}}{2}(P+Q)\right] + \left[\frac{\sqrt{2}}{2}(P+Q)\right]^2 - 3 = 0$$

$$\frac{2}{4}(P-Q)^2 + 16 \cdot \frac{2}{4}(P-Q)(P+Q) + \frac{2}{4}(P+Q)^2 - 3 = 0$$

$$\frac{1}{2}(P-Q)(P-Q) + 8(P^2 - Q^2) + \frac{1}{2}(P+Q)(P+Q) - 3 = 0$$

$$\frac{1}{2}(P^2 - PQ - PQ + Q^2) + 8P^2 - 8Q^2 + \frac{1}{2}(P^2 + PQ + PQ + Q^2) - 3 = 0$$

$$\frac{1}{2}(P^2 - 2PQ + Q^2) + 8P^2 - 8Q^2 + \frac{1}{2}(P^2 + 2PQ + Q^2) - 3 = 0$$

$$\frac{1}{2}P^2 - PQ + \frac{1}{2}Q^2 + 8P^2 - 8Q^2 + \frac{1}{2}P^2 + PQ + \frac{1}{2}Q^2 - 3 = 0$$

$$9P^2 - 7Q^2 - 3 = 0$$

$$9P^2 - 7Q^2 = 3$$

LET $P = x'$ AND $Q = y'$

$$9x'^2 - 7y'^2 = 3$$