

6.

a) $\sin(\alpha + \beta)$

$$= \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= \left(\frac{\sqrt{74}}{9}\right)\left(\frac{\sqrt{5}}{3}\right) + \left(\frac{\sqrt{7}}{9}\right)\left(-\frac{2}{3}\right)$$

$$= \frac{\sqrt{370}}{27} - \frac{2\sqrt{7}}{27}$$

$$= \frac{\sqrt{370} - 2\sqrt{7}}{27}$$

b) $\cos(\alpha + \beta)$

$$= \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \left(\frac{\sqrt{7}}{9}\right)\left(\frac{\sqrt{5}}{3}\right) - \left(\frac{\sqrt{74}}{9}\right)\left(-\frac{2}{3}\right)$$

$$= \frac{\sqrt{35}}{27} + \frac{2\sqrt{74}}{27}$$

$$= \frac{\sqrt{35} + 2\sqrt{74}}{27}$$

c) $\sin(\alpha - \beta)$

$$= \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$= \left(\frac{\sqrt{74}}{9}\right)\left(\frac{\sqrt{5}}{3}\right) - \left(\frac{\sqrt{7}}{9}\right)\left(-\frac{2}{3}\right)$$

$$= \frac{\sqrt{370}}{27} + \frac{2\sqrt{7}}{27}$$

$$= \frac{\sqrt{370} + 2\sqrt{7}}{27}$$

d) $\tan(\alpha - \beta)$

$$= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$= \frac{\frac{\sqrt{518}}{7} - \left(-\frac{2\sqrt{5}}{5}\right)}{1 + \frac{\sqrt{518}}{7} \left(-\frac{2\sqrt{5}}{5}\right)}$$

$$= \frac{\frac{\sqrt{518}}{7} + \frac{2\sqrt{5}}{5}}{1 - \frac{2\sqrt{2590}}{35}}$$

$$= \frac{5\sqrt{518} + 14\sqrt{5}}{35 - 2\sqrt{2590}}$$

$$= \frac{35(1) - 35\left(\frac{2\sqrt{2590}}{35}\right)}{35 - 2\sqrt{2590}}$$

$$= \frac{5\sqrt{518} + 14\sqrt{5}}{35 - 2\sqrt{2590}}$$

$$\frac{5\sqrt{518} + 14\sqrt{5}}{35 - 2\sqrt{2590}}, \frac{35 + 2\sqrt{2590}}{35 + 2\sqrt{2590}}$$

$$\frac{175\sqrt{518} + 10\sqrt{1341620} + 490\sqrt{5} + 28\sqrt{12950}}{1225 - 10360}$$