11.

$$\sec\left(\sin^{-1}(u-3)+\tan^{-1}(u-5)\right)$$

$$=\frac{1}{\cos\left(\underbrace{\sin^{-1}(u-3)}_{\alpha}+\underbrace{\tan^{-1}(u-5)}_{\beta}\right)}$$

$$\boxed{\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$

$$=\frac{1}{\underbrace{\cos\left(\sin^{-1}(u-3)\right)}_{①}\underbrace{\cos\left(\tan^{-1}(u-5)\right)}_{②}-\underbrace{\sin\left(\sin^{-1}(u-3)\right)}_{③}\underbrace{\sin\left(\tan^{-1}(u-5)\right)}_{④}}$$

$$=\frac{1}{\underbrace{\sqrt{1-(u-3)^2}}_{①}\underbrace{\left(\frac{1}{\sqrt{1+(u-5)^2}}\right)}_{②}-\underbrace{(u-3)}_{③}\underbrace{\left(\frac{u-5}{\sqrt{1+(u-5)^2}}\right)}_{④}}$$

$$=\frac{\sqrt{1+(u-5)^2}}{\sqrt{1-(u-3)^2}-(u-3)(u-5)}$$

① $\cos\left(\sin^{-1}(u-3)\right)$

$p=\sin^{-1}(u-3)$

$-1\le u-3\le 1$ $\quad \sin p=\frac{u-3}{1}$

$2\le u\le 4$ $\quad$ so $y=u-3$ $\quad r=1$

$x^2+y^2=r^2$

$x^2+(u-3)^2=1^2$

$x^2=1-(u-3)^2$

$x=\sqrt{1-(u-3)^2}$

$\cos=\frac{x}{r}=\frac{\sqrt{1-(u-3)^2}}{1}$

② $\cos\left(\tan^{-1}(u-5)\right)$

$p=\tan^{-1}(u-5)$

$\tan p=\frac{u-5}{1}$

so $y=u-5$ $\quad x=1$

$r^2=x^2+y^2$

$r=\sqrt{1^2+(u-5)^2}$

$r=\sqrt{1+(u-5)^2}$

$\cos=\frac{x}{r}=\frac{1}{\sqrt{1+(u-5)^2}}$

④ $\sin\left(\tan^{-1}(u-5)\right)$

$\sin=\frac{y}{r}$

$=\frac{u-5}{\sqrt{1+(u-5)^2}}$