6. $f(x) = x\sqrt{4+x}$

① $f(x) = \underbrace{x}_{P}\underbrace{(4+x)^{1/2}}_{Q}$

$P' = 1 \quad Q' = \frac{1}{2}(4+x)^{-\frac{1}{2}}$

$Q' = \frac{1}{2(4+x)^{1/2}}$

$P'Q + PQ'$

$f'(x) = 1(4+x)^{1/2} + x\left(\frac{1}{2(4+x)^{1/2}}\right)$

$= (4+x)^{1/2} + \frac{x}{2(4+x)^{1/2}}$

$= \frac{(4+x)^{\frac{1}{2}} \cdot 2(4+x)^{\frac{1}{2}}}{2(4+x)^{1/2}} + \frac{x}{2(4+x)^{1/2}}$

$= \frac{2(4+x)^{1} + x}{2(4+x)^{1/2}}$

$= \frac{8 + 2x + x}{2(4+x)^{1/2}}$

$= \frac{3x+8}{2(4+x)^{1/2}}$ ) P
) Q

$P' = 3 \quad Q' = 2 \cdot \frac{1}{2}(4+x)^{-\frac{1}{2}} \cdot 1$

$Q' = \frac{1}{(4+x)^{1/2}}$

$\frac{P'Q - PQ'}{Q^2}$

$f''(x) = \frac{3 \cdot 2(4+x)^{1/2} - (3x+8)\left(\frac{1}{(4+x)^{1/2}}\right)}{\left[2(4+x)^{1/2}\right]^2}$

$= \frac{6(4+x)^{1/2} - \frac{3x+8}{(4+x)^{1/2}}}{4\left[(4+x)^{1/2}\right]^2}$

$= \frac{6(4+x)^{1/2} - \frac{3x+8}{(4+x)^{1/2}}}{4(4+x)^{1}}$

$= \frac{6(4+x)^{\frac{1}{2}}(4+x)^{\frac{1}{2}} - \frac{3x+8}{(4+x)^{1/2}} \cdot (4+x)^{\frac{1}{2}}}{4(4+x)^{1} \cdot (4+x)^{1/2}}$

$= \frac{6(4+x)^{1} - (3x+8)}{4(4+x)^{3/2}}$

$= \frac{24 + 6x - 3x - 8}{4(4+x)^{3/2}}$

$f''(x) = \frac{3x+16}{4(4+x)^{3/2}}$