

Evaluating Limits Analytically

1. Use a graphing utility to graph the function and visually estimate the limits

$$f(x) = x|x - 3|$$

$$(a) \lim_{x \rightarrow 3} f(x)$$

$$(b) \lim_{x \rightarrow -1} f(x)$$

2. Find the limit

$$\lim_{x \rightarrow -3} (x^2 - 3x)$$

3. Find the limit

$$\lim_{x \rightarrow -1} \left(\frac{4x - 1}{3 - x} \right)$$

4. Find the limit

$$\lim_{x \rightarrow 4} \left(\frac{\frac{1}{x+3} - \frac{1}{3}}{x} \right)$$

5. Find the limit

$$\lim_{x \rightarrow \frac{\pi}{3}} (\tan x)$$

6. Find the limit

$$\lim_{x \rightarrow 9} \left(\csc \frac{\pi x}{6} \right)$$

7. Find the limit

$$f(x) = x + 3, g(x) = x^2 - 1$$

$$(a) \lim_{x \rightarrow 2} (f(x))$$

$$(b) \lim_{x \rightarrow 3} (g(x))$$

$$(c) \lim_{x \rightarrow 2} (g(f(x)))$$

8. Use the information to evaluate the limits

$$\lim_{x \rightarrow c} f(x) = 8$$

$$\lim_{x \rightarrow c} g(x) = 4$$

$$(a) \lim_{x \rightarrow c} [5f(x)]$$

$$(b) \lim_{x \rightarrow c} [f(x) + g(x)]$$

$$(c) \lim_{x \rightarrow c} [f(x)g(x)]$$

$$(d) \lim_{x \rightarrow c} \left[\frac{f(x)}{g(x)} \right]$$

9. Use the graph to determine the limit visually (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$f(x) = \frac{x}{x^2 - 2x}$$

$$(a) \lim_{x \rightarrow 3} f(x)$$

$$(b) \lim_{x \rightarrow 0} f(x)$$

10. Find the limit of the function (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$\lim_{x \rightarrow 4} \left(\frac{2x^2 - 9x + 4}{x - 4} \right)$$

11. Find the limit of the function (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$\lim_{x \rightarrow -3} \left(\frac{x^2 - 4x - 21}{x^2 - 9} \right)$$

12. Find the limit (if it exists)

$$\lim_{x \rightarrow 7} \left(\frac{\sqrt{x+2} - 3}{x-7} \right)$$

13. Find the limit (if it exists)

$$\lim_{x \rightarrow 0} \left(\frac{\frac{1}{x+6} - \frac{1}{6}}{x} \right)$$

14. Find the limit (if it exists)

$$\lim_{\Delta x \rightarrow 0} \left(\frac{(x + \Delta x)^2 + 2(x + \Delta x) - (x^2 + 2x)}{\Delta x} \right)$$

15. Determine the limit of the transcendental function (if it exists)

$$\lim_{x \rightarrow 0} \left(\frac{9}{x} - \frac{9 \cos x}{x} \right)$$

16. Determine the limit of the transcendental function (if it exists)

$$\lim_{\theta \rightarrow 0} \left(\frac{1}{\theta \csc \theta} \right)$$

17. Determine the limit of the transcendental function (if it exists)

$$\lim_{x \rightarrow 0} \left(\frac{5}{x \cot^2 x} \right)$$

18. Determine the limit of the transcendental function (if it exists)

$$\lim_{x \rightarrow 0} \left(\frac{5(e^{2x} - 1)}{e^x - 1} \right)$$

19. (similar to p.88 #91-94)

$$\text{Find : } \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

for :

$$f(x) = \sqrt{x-3}$$

20. Use the Squeeze Theorem to find the limit
(similar to p.88 #95-96)

$$\lim_{x \rightarrow c} f(x)$$

where

$$c = 0$$

$$3 - |x| \leq f(x) \leq 3 + |x|$$