13. $\lim_{x\to 0} \dfrac{\frac{1}{x+6} - \frac{1}{6}}{x}$

$\lim_{x\to 0} \dfrac{6(x+6)\left(\frac{1}{x+6}\right) - \frac{1}{6}\cdot 6(x+6)}{x\cdot 6(x+6)}$

$\lim_{x\to 0} \dfrac{6 - 1(x+6)}{6x(x+6)}$

$\lim_{x\to 0} \dfrac{6 - x - 6}{6x(x+6)}$

$\lim_{x\to 0} \dfrac{-x}{6x(x+6)}$

$\lim_{x\to 0} \dfrac{-1}{6(x+6)}$

$= \dfrac{-1}{6(0+6)}$

$= \boxed{\dfrac{-1}{36}}$

14. $\lim_{\Delta x\to 0}\left(\dfrac{(x+\Delta x)^2 + 2(x+\Delta x) - (x^2+2x)}{\Delta x}\right)$

$= \lim_{\Delta x\to 0} \dfrac{(x+\Delta x)(x+\Delta x) + 2x + 2\Delta x - x^2 - 2x}{\Delta x}$

$= \lim_{\Delta x\to 0} \dfrac{x^2 + x\Delta x + x\Delta x + (\Delta x)^2 + 2\Delta x - x^2}{\Delta x}$

$= \lim_{\Delta x\to 0} \dfrac{2x\Delta x + (\Delta x)^2 + 2\Delta x}{\Delta x}$

$= \lim_{\Delta x\to 0} \dfrac{\Delta x[2x + \Delta x + 2]}{\Delta x}$

$= \lim_{\Delta x\to 0} (2x + \Delta x + 2)$

$= 2x + 0 + 2$

$= \boxed{2x+2}$

SPECIAL FORMULAS

1. $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$
2. $\lim_{x\to 0} \dfrac{1-\cos x}{x} = 0$
3. $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

15. $\lim_{x\to 0}\left(\dfrac{9}{x} - \dfrac{9\cos x}{x}\right)$

$= \lim_{x\to 0}\left(\dfrac{9 - 9\cos x}{x}\right)$

$= \lim_{x\to 0}\left[\dfrac{9(1-\cos x)}{x}\right]$

$= 9\lim_{x\to 0} \dfrac{1-\cos x}{x}$

$= 9(0)$

$= \boxed{0}$

16. $\lim_{\theta\to 0} \dfrac{1}{\theta\csc\theta}$

$= \lim_{\theta\to 0} \dfrac{1}{\theta\left(\frac{1}{\sin\theta}\right)}$

$= \lim_{\theta\to 0} \dfrac{\sin\theta}{\theta}$

$= \boxed{1}$