4. $\int x^2 (x^3+1)^3 \, dx$

$u = x^3+1 \quad du = 3x^2 \, dx$

$= \frac{1}{3} \int 3x^2 (x^3+1)^3 \, dx$

$= \frac{1}{3} \int u^3 \, du$

$= \frac{1}{3} \cdot \frac{1}{4} u^4 + C$

$= \frac{1}{12} (x^3+1)^4 + C$

5. $\int t^4 \sqrt{t^5+3} \, dt$

$u = t^5+3 \quad du = 5t^4 \, dt$

$= \frac{1}{5} \int 5t^4 (t^5+3)^{\frac{1}{2}} \, dt$

$= \frac{1}{5} \int u^{\frac{1}{2}} \, du$

$= \frac{1}{5} \cdot \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= \frac{1}{5} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{1}{5} \cdot \frac{2}{3} u^{\frac{3}{2}} + C$

$= \frac{2}{15} (t^5+3)^{\frac{3}{2}} + C$

6. $\int \frac{x^2}{(5x^3-3)^2} \, dx$

$u = 5x^3-3 \quad du = 15x^2 \, dx$

$= \frac{1}{15} \int \frac{15x^2}{(5x^3-3)^2} \, dx$

$= \frac{1}{15} \int \frac{1}{u^2} \, du$

$= \frac{1}{15} \int u^{-2} \, du$

$= \frac{1}{15} \cdot \frac{u^{-2+1}}{-2+1} + C$

$= \frac{1}{15} \cdot \frac{u^{-1}}{-1} + C$

$= \frac{1}{-15u} + C$

$= \frac{-1}{15(5x^3-3)} + C$