19. $\int x\sqrt{3x+2}\,dx$

$u = 3x+2 \quad du = 3\,dx$

$u-2 = 3x$

$\frac{u-2}{3} = x$

$\frac{1}{3}\int 3x\sqrt{3x+2}\,dx$

$\frac{1}{3}\int \frac{u-2}{3}\sqrt{u}\,du$

$\frac{1}{3}\cdot\frac{1}{3}\int (u-2)u^{\frac{1}{2}}\,du$

$\frac{1}{9}\int \left(u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right)du$

$= \frac{1}{9}\left[\frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1} - 2\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]+C$

$= \frac{1}{9}\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - 2\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]+C$

$= \frac{1}{9}\left[\frac{2}{5}u^{\frac{5}{2}} - 2\cdot\frac{2}{3}u^{\frac{3}{2}}\right]+C$

$= \frac{1}{9}\left[\frac{2}{5}(3x+2)^{\frac{5}{2}} - \frac{4}{3}(3x+2)^{\frac{3}{2}}\right]+C$

$= \boxed{\frac{2}{45}(3x+2)^{\frac{5}{2}} - \frac{4}{27}(3x+2)^{\frac{3}{2}}+C}$

20. $\int \frac{4x+2}{\sqrt{x+2}}\,dx$

$u = x+2 \quad du = dx$

$u-2 = x$

$= \int \frac{4(u-2)+2}{\sqrt{u}}\,du$

$= \int \frac{4u-8+2}{u^{\frac{1}{2}}}\,du$

$= \int \frac{4u-6}{u^{\frac{1}{2}}}\,du$

$\int \frac{4u^1}{u^{\frac{1}{2}}} - \frac{6}{u^{\frac{1}{2}}}\,du$

$= \int \left(4u^{\frac{1}{2}} - 6u^{-\frac{1}{2}}\right)du$

$= 4\cdot\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 6\cdot\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C$

$= 4\cdot\frac{u^{3/2}}{\frac{3}{2}} - 6\cdot\frac{u^{1/2}}{\frac{1}{2}}+C$

$= 4\cdot\frac{2}{3}u^{3/2} - 6\cdot\frac{2}{1}u^{1/2}+C$

$= \boxed{\frac{8}{3}(x+2)^{3/2} - 12(x+2)^{1/2}+C}$