6. $\int \frac{x^4-9}{x^2+3}\,dx$

$= \int \frac{(x^2+3)(x^2-3)}{x^2+3}\,dx$

$= \int (x^2-3)\,dx$

$= \frac{1}{3}x^3 - 3x + C$

7. $\int \frac{5}{\sqrt{x}(2+x)}\,dx$

$\int \frac{5}{\sqrt{x}\left[(\sqrt{2})^2+(\sqrt{x})^2\right]}\,dx$

$a=\sqrt{2} \quad u=\sqrt{x}=x^{\frac{1}{2}} \quad du=\frac{1}{2}x^{-\frac{1}{2}}\,dx$

$du=\frac{1}{2x^{1/2}}\,dx$

$du=\frac{1}{2\sqrt{x}}\,dx$

$= 5\cdot 2\int \frac{1}{2\sqrt{x}\left[(\sqrt{2})^2+(\sqrt{x})^2\right]}\,dx$

$= 10\int \frac{1}{a^2+u^2}\,du$

$= 10\cdot\frac{1}{a}\arctan\frac{u}{a}+C$

$= 10\cdot\frac{1}{\sqrt{2}}\arctan\frac{\sqrt{x}}{\sqrt{2}}+C$

$= \frac{10}{\sqrt{2}}\arctan\frac{\sqrt{x}}{\sqrt{2}}+C$