10. $\int \frac{1}{x^2-6x+50} \, dx$

$(-6 \cdot \frac{1}{2})^2$

$(-3)^2$

$\underline{\underline{9}}$

$= \int \frac{1}{x^2-6x+9 \underset{\underline{\quad}}{} -9 +50} \, dx$

$= \int \frac{1}{(x-3)^2 + 41} \, dx$

$= \int \frac{1}{41+(x-3)^2} \, dx$

$= \int \frac{1}{(\sqrt{41})^2 + (x-3)^2} \, dx$

$a = \sqrt{41} \quad u = x-3 \quad du = 1 \cdot dx$

$= \int \frac{1}{a^2+u^2} \, du$

$= \frac{1}{a} \operatorname{arcTAN} \frac{u}{a} + C$

$= \frac{1}{\sqrt{41}} \operatorname{arcTAN} \frac{x-3}{\sqrt{41}} + C$

$= \boxed{\frac{\sqrt{41}}{41} - \operatorname{arcTAN}\left[\frac{\sqrt{41}}{41} (x-3)\right] + C}$

11. $\int \frac{2}{\sqrt{3-8x-x^2}} \, dx$

$= \int \frac{2}{\sqrt{-x^2-8x+3}} \, dx$

$= \int \frac{2}{\sqrt{-1(x^2+8x)+3}} \, dx$

$(8 \cdot \frac{1}{2})^2$

$(4)^2$

16

$= \int \frac{2}{\sqrt{-1(x^2+8x+16 -16)+3}} \, dx$

$= \int \frac{2}{\sqrt{-1(x^2+8x+16)-1(-16)+3}} \, dx$

$= \int \frac{2}{\sqrt{-1(x+4)^2 +16+3}} \, dx$

$= \int \frac{2}{\sqrt{-1(x+4)^2 +19}} \, dx$

$= \int \frac{2}{\sqrt{-(x+4)^2+19}} \, dx$

$= \int \frac{2}{\sqrt{19-(x+4)^2}} \, dx$

$\overset{?}{=} 2 \int \frac{1}{\sqrt{(\sqrt{19})^2 - (x+4)^2}} \, dx$

$a = \sqrt{19} \quad u = x+4 \quad du = 1 \cdot dx$

$= 2 \int \frac{1}{\sqrt{a^2-u^2}} \, du$

$= 2 \operatorname{arcsin} \frac{u}{a} + C$

$= 2 \operatorname{arcsin} \frac{x+4}{\sqrt{19}} + C$

$= \boxed{2 \operatorname{arcsin}\left[\frac{\sqrt{19}}{19} (x+4)\right] + C}$