

$$5. \int \cos^4(4x) dx$$

RECALL

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\int \cos^2(\underline{4x}) \cos^2(4x) dx$$

$$\int \left( \frac{1 + \cos 2 \cdot 4x}{2} \right) \left( \frac{1 + \cos 2 \cdot 4x}{2} \right) dx$$

$$\frac{1}{4} \int (1 + \cos 8x)(1 + \cos 8x) dx$$

$$\frac{1}{4} \int (1 + \cos 8x + \cos 8x + \cos^2 8x) dx$$

$$\frac{1}{4} \int (1 + 2\cos 8x + \cos^2 \underline{8x}) dx$$

$$\frac{1}{4} \int \left( 1 + 2\cos 8x + \frac{1 + \cos 2 \cdot 8x}{2} \right) dx$$

$$\frac{1}{4} \int \left( 1 + 2\cos 8x + \frac{1 + \cos 16x}{2} \right) dx$$

$$\frac{1}{4} \int \left( 1 + 2\cos 8x + \frac{1}{2} + \frac{1}{2}\cos 16x \right) dx$$

$$\frac{1}{4} \int \left( \frac{3}{2} + 2\cos 8x + \frac{1}{2}\cos 16x \right) dx$$

$$\frac{1}{4} \left[ \int \frac{3}{2} dx + 2 \int \cos \frac{8x}{u} dx + \frac{1}{2} \int \cos \frac{16x}{w} dx \right]$$

$du = 8 dx \qquad dw = 16 dx$

$$\frac{1}{4} \left[ \frac{3}{2}x + 2 \cdot \frac{1}{8} \int 8 \cos 8x dx + \frac{1}{2} \cdot \frac{1}{16} \int 16 \cos 16x dx \right]$$

$$\frac{1}{4} \left[ \frac{3}{2}x + \frac{1}{4} \int \cos u du + \frac{1}{32} \int \cos w dw \right]$$

$$\frac{1}{4} \left[ \frac{3}{2}x + \frac{1}{4} \sin u + \frac{1}{32} \sin w \right] + C$$

$$\frac{3}{8}x + \frac{1}{16} \sin 8x + \frac{1}{128} \sin 16x + C$$