13. $\int \cot^9 x \csc^2 x \, dx$

SAVE $\csc^2 x$

$\int (\cot x)^9 \csc^2 x \, dx$

$u = \cot x \quad du = -\csc^2 x \, dx$

$-\int (\cot x)^9 (-\csc^2 x) \, dx$

$-\int u^9 \, du$

$-\frac{1}{10} u^{10} + C$

$-\frac{1}{10} \cot^{10} x + C$

14. $\int \cot^5 x \csc^3 x \, dx$

RECALL

$1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$\int \cot^4 x \csc^2 x \, \csc x \cot x \, dx$

SAVE $\csc x \cot x$

$\int \cot^2 x \cot^2 x \csc^2 x \, \csc x \cot x \, dx$

$\int (\csc^2 x - 1)(\csc^2 x - 1) \csc^2 x \, \csc x \cot x \, dx$

$\int (\csc^6 x - 2\csc^4 x + \csc^2 x) \csc x \cot x \, dx$

$\int \left[(\csc x)^6 - 2(\csc x)^4 + (\csc x)^2 \right] \csc x \cot x \, dx$

$u = \csc x \quad du = -\csc x \cot x \, dx$

$-\int (u^6 - 2u^4 + u^2) \, du$

$-\left[\frac{1}{7} u^7 - \frac{2}{5} u^5 + \frac{1}{3} u^3 \right] + C$

$-\frac{1}{7} \csc^7 x + \frac{2}{5} \csc^5 x - \frac{1}{3} \csc^3 x + C$