15. $\int \frac{\cot^5 x}{\csc x}\,dx$

RECALL

$1+\cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$\int \frac{\cot^2 x \cot^2 x \cot x}{\csc x}\,dx$

$\int \frac{(\csc^2-1)(\csc^2-1)}{\csc} \underbrace{\cot}_{\text{SAVE}}\,dx$

$\int \frac{\csc^4 - 2\csc^2 + 1}{\csc} \underline{\cot}\,dx$

$\int \left(\frac{\csc^4}{\csc} - \frac{2\csc^2}{\csc} + \frac{1}{\csc}\right)\cot\,dx$

$\int \left[\csc^3 - 2\csc + \frac{1}{\csc}\right]\cot\,dx$

$\int \csc^3 \cot\,dx - 2\int \csc\cot\,dx + \int \frac{\cot}{\csc}\,dx$

$\int (\csc x)^2 \underbrace{\csc x \cot x}\,dx - 2(-\csc x) + \int \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}\,dx$

$u = \csc x \quad du = -\csc x \cot x\,dx$

$-\int u^2\,du + 2\csc x + \int \cos x\,dx$

$-\frac{1}{3}u^3 + 2\csc x + \sin x + C$

$\boxed{-\frac{1}{3}\csc^3 x + 2\csc x + \sin x + C}$