Form: $y' + P(x)y = Q(x)$

7. $4y' + y = 5x$

① $\frac{4y'}{4} + \frac{y}{4} = \frac{5x}{4}$

$y' + \underbrace{\frac{1}{4}}_{P(x)} y = \frac{5}{4}x$

② I.F.

$M(x) = e^{\int P(x)\,dx}$

$= e^{\int \frac{1}{4}\,dx}$

$= e^{\frac{1}{4}x}$

③ $e^{\frac{1}{4}x}(y' + \frac{1}{4}y) = e^{\frac{1}{4}x}(\frac{5}{4}x)$

$\underbrace{e^{\frac{1}{4}x}}_{Q}\underbrace{y'}_{P'} + \underbrace{\frac{1}{4}e^{\frac{1}{4}x}}_{Q'}\underbrace{y}_{P} = \frac{5}{4}xe^{\frac{1}{4}x}$

④ $\frac{d}{dx}[ye^{\frac{1}{4}x}] = \frac{5}{4}xe^{\frac{1}{4}x}$

⑤ $\int \frac{d}{dx}[ye^{\frac{1}{4}x}]\,dx = \int (\frac{5}{4}xe^{\frac{1}{4}x})\,dx$ "PARTS"

S	D	I
+ →	$\frac{5}{4}x$	$e^{\frac{1}{4}x}$
− →	$\frac{5}{4}$	$4e^{\frac{1}{4}x}$
+	0	$16e^{\frac{1}{4}x}$

$ye^{\frac{1}{4}x} = 5xe^{\frac{1}{4}x} - 20e^{\frac{1}{4}x} + C$

$\frac{ye^{\frac{1}{4}x}}{e^{\frac{1}{4}x}} = \frac{5xe^{\frac{1}{4}x}}{e^{\frac{1}{4}x}} - \frac{20e^{\frac{1}{4}x}}{e^{\frac{1}{4}x}} + \frac{C}{e^{\frac{1}{4}x}}$

$y = 5x - 20 + Ce^{-\frac{1}{4}x}$