Form: $y' + P(x)y = Q(x)$

11. $y' + 4y = xe^{-4x}$ $\quad y(1) = 0$

(3) $e^{4x}(y' + 4y) = e^{4x} \cdot xe^{-4x}$

(4) $\underbrace{e^{4x}}_{Q}\underbrace{y'}_{P'} + \underbrace{4e^{4x}}_{Q'}\underbrace{y}_{P} = x$

$\frac{d}{dx}[y e^{4x}] = x$

(5) $\int \frac{d}{dx}[y e^{4x}]\,dx = \int x\,dx$

$y e^{4x} = \frac{1}{2}x^2 + C$

$y = \frac{x^2}{2e^{4x}} + \frac{C}{e^{4x}}$

$y = \frac{x^2}{2e^{4x}} - \frac{\frac{1}{2}}{e^{4x}}$

$$\boxed{y = \frac{x^2}{2e^{4x}} - \frac{1}{2e^{4x}}}$$

(2) I.F.

$\mu(x) = e^{\int P(x)\,dx}$

$= e^{\int 4\,dx}$

$= e^{4x}$

$y(1) = 0$

$x = 1 \quad y = 0$

$0 = \frac{1^2}{2e^{4\cdot 1}} + \frac{C}{e^{4\cdot 1}}$

$0 = \frac{1}{2e^4} + \frac{C}{e^4}$

$0 = \frac{1}{2} + C$

$-\frac{1}{2} = C$