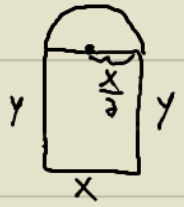
4.



AREA = AREA of RECTANGLE + AREA of HALF CIRCLE

$A = xy + \frac{1}{2}\pi r^2$

$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$A = xy + \frac{1}{8}\pi x^2$

PERIMETER $= x + 2y + \pi\left(\frac{x}{2}\right)$

$P = x + 2y + \frac{\pi}{2}x$

$20 = x + 2y + \frac{\pi}{2}x$

$40 = 2x + 4y + \pi x$

$40 - 2x - \pi x = 4y$

$10 - \frac{2}{4}x - \frac{\pi}{4}x = y$

$10 - \frac{1}{2}x - \frac{\pi}{4}x = y$

CIRCLE	$\frac{1}{2}$ CIRCLE
$A = \pi r^2$	$A = \frac{1}{2}\pi r^2$
$C = 2\pi r$	$C = \frac{1}{2}\cdot 2\pi r$
	$= \pi r$

MAXIMIZE AREA

$A = xy + \frac{1}{8}\pi x^2$

$A = x\left(10 - \frac{1}{2}x - \frac{\pi}{4}x\right) + \frac{1}{8}\pi x^2$

$A = 10x - \frac{1}{2}x^2 - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2$

$A = 10x - \frac{1}{2}x^2 - \frac{2\pi}{8}x^2 + \frac{\pi}{8}x^2$

$A = 10x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2$

$\frac{dA}{dx} = 10 - x - \frac{2\pi}{8}x$

$= \boxed{10 - x - \frac{\pi}{4}x}$

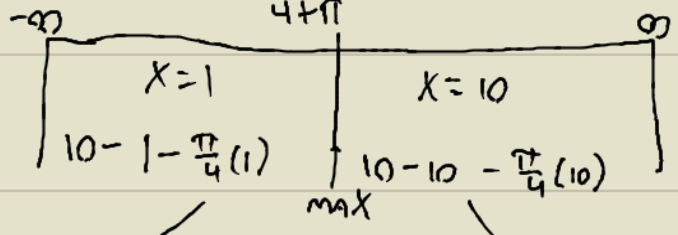
$10 - x - \frac{\pi}{4}x = 0$

$40 - 4x - \pi x = 0$

$40 = 4x + \pi x$

$40 = x(4+\pi)$

$\frac{40}{4+\pi} = x$



MAX AT $\boxed{x = \frac{40}{4+\pi}}$

$y = 10 - \frac{1}{2}x - \frac{\pi}{4}x$

$y = 10 - \frac{1}{2}\left(\frac{40}{4+\pi}\right) - \frac{\pi}{4}\left(\frac{40}{4+\pi}\right)$

$= 10 - \frac{20}{4+\pi} - \frac{10\pi}{4+\pi}$

$y = 10 - \frac{20+10\pi}{4+\pi}$

$y = \frac{10}{1} - \frac{20+10\pi}{4+\pi}$

$= \frac{40+10\pi}{4+\pi} - \frac{20+10\pi}{4+\pi}$

$y = \frac{40+10\pi-20-10\pi}{4+\pi}$

$\boxed{y = \frac{20}{4+\pi}}$