4. $f(x) = \dfrac{x+5}{x^2+7x+10}$ $\quad P \quad P' = 1$, $\quad Q \quad Q' = 2x+7$

$\boxed{\text{HARD WAY}}$

$$\frac{P'Q - PQ'}{Q^2}$$

$$f'(x) = \frac{1(x^2+7x+10) - (x+5)(2x+7)}{(x^2+7x+10)^2}$$

$$= \frac{(x+2)(x+5) - (x+5)(2x+7)}{[(x+2)(x+5)]^2}$$

$$= \frac{\cancel{(x+5)}[x+2-(2x+7)]}{(x+2)\cancel{(x+5)}(x+2)(x+5)}$$

$$= \frac{x+2-2x-7}{(x+2)(x+2)(x+5)}$$

$$= \frac{-x-5}{(x+2)(x+2)(x+5)}$$

$$= \frac{-1\cancel{(x+5)}}{(x+2)(x+2)\cancel{(x+5)}}$$

$$= \boxed{\frac{-1}{(x+2)^2}}$$

$\boxed{\text{EASIER WAY}}$

$$f(x) = \frac{\cancel{x+5}}{(x+2)\cancel{(x+5)}} \quad \text{PSD}$$

$f(x) = \dfrac{1}{x+2}$ $\quad P \quad P' = 0$, $\quad Q \quad Q' = 1$

$$\frac{P'Q - PQ'}{Q^2}$$

$$f'(x) = \frac{0(x+2) - 1(1)}{(x+2)^2}$$

$$= \boxed{\frac{-1}{(x+2)^2}}$$