DERIVATIVE - LIMIT DEFINITION

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

2. $f(x) = x^2 + 3x - 1$

① $f(x) = x^2 + 3x - 1$

② $f(x+h) = (x+h)^2 + 3(x+h) - 1$

$= (x+h)(x+h) + 3x + 3h - 1$

$= x^2 + hx + hx + h^2 + 3x + 3h - 1$

$f(x+h) = x^2 + 2hx + h^2 + 3x + 3h - 1$

③ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + 3x + 3h - 1 - (x^2 + 3x - 1)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + 3x + 3h - 1 - x^2 - 3x + 1}{h}$

$= \lim_{h \to 0} \frac{2hx + h^2 + 3h}{h}$

$= \lim_{h \to 0} \frac{h(2x + h + 3)}{h}$

$= \lim_{h \to 0} 2x + h + 3$

$= 2x + 0 + 3$

$f'(x) = 2x + 3$