4. $f(x) = \frac{1}{x-3}$

① $f(x) = \frac{1}{x-3}$

② $f(x+h) = \frac{1}{(x+h)-3} = \frac{1}{x+h-3}$

③ $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h-3} - \frac{1}{x-3}}{h}$$

$$= \lim_{h\to 0} \frac{(x-3)(x+h-3)\left(\frac{1}{x+h-3}\right) - \frac{1}{x-3}(x-3)(x+h-3)}{h(x-3)(x+h-3)}$$

$$= \lim_{h\to 0} \frac{x-3-(x+h-3)}{h(x-3)(x+h-3)}$$

$$= \lim_{h\to 0} \frac{x-3-x-h+3}{h(x-3)(x+h-3)}$$

$$= \lim_{h\to 0} \frac{-h}{h(x-3)(x+h-3)}$$

$$= \lim_{h\to 0} \frac{-1}{(x-3)(x+h-3)}$$

$$= \frac{-1}{(x-3)(x+0-3)}$$

$$= \frac{-1}{(x-3)(x-3)}$$

$$= \boxed{\frac{-1}{(x-3)^2}}$$