

Multiplicative Inverses of
Matrices and Matrix Equations

1. Find the products AB and BA to determine whether B is a multiplicative inverse of A

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 7 & -1 \\ 5 & 9 \end{bmatrix}$$

2. Find the products AB and BA to determine whether B is a multiplicative inverse of A

$$A = \begin{bmatrix} 7 & -4 \\ -2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} \frac{3}{13} & \frac{4}{13} \\ \frac{2}{13} & \frac{7}{13} \end{bmatrix}$$

3. Use the following fact:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then } A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

to find the inverse of each matrix, if possible.
Check that the matrix you find is the inverse.

$$A = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$$

5. Find A^{-1} by forming $[A|I]$ and then using row operations to obtain $[I|B]$ where $A^{-1} = [B]$. Check that $AA^{-1} = I$ and $A^{-1}A = I$

$$A = \begin{bmatrix} 3 & -1 \\ 4 & 5 \end{bmatrix}$$

6. Write each linear system as a matrix equation in the form $AX = B$ where A is the coefficient matrix and B is the constant matrix

$$3x + 7y = 10$$

$$4x - 2y = 9$$

8. Write each matrix equation as a system of linear equations without matrices

$$\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -7 \end{bmatrix}$$

9. a) Write each linear system as a matrix equation in the form $AX = B$.
b) Solve the system using the inverse that is given for the coefficient matrix.

$$\begin{array}{l} 3x - 5y + z = -1 \\ x - 3y - 2z = 3 \\ 4x + y + 3z = 3 \end{array} \quad \text{the inverse of} \quad \begin{bmatrix} 3 & -5 & 1 \\ 1 & -3 & -2 \\ 4 & 1 & 3 \end{bmatrix} \text{ is } \begin{bmatrix} -7/47 & 16/47 & 13/47 \\ -11/47 & 5/47 & 7/47 \\ 13/47 & -23/47 & -4/47 \end{bmatrix}$$