Binomial Probability Distributions

Def: Binomial probability distribution

1. procedure has fixed # of trials
2. trials must be independent
3. each trial must have all outcomes classified into 2 categories
4. probabilities must remain constant for each trial

1. Determine which of the following probability experiments represents a binomial experiment. If the probability experiment is not binomial, state why (Similar to p.344 #7-16)

Twenty students are asked how many tacos they eat each month.

2. Determine which of the following probability experiments represents a binomial experiment. If the probability experiment is not binomial, state why (Similar to p.344 #7-16)

Fifty people are asked on whether they believe in reincarnation.

Notation:

- P(S) = p (p=probability of a success in one trial)
- P(F) = 1-p = q (q=probability of a failure in one trial)
- n= # of trials
- x = # of successes in n trials
- P(x) = probability of getting exactly x successes among the n trials

Binomial probability formula (for x = 0, 1, 2, …., n):

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x (1-p)^{n-x}$$

3. Finding Probabilities (By Formula)

n = 10, p=0.3, find the P(x = 4)

4. Finding Probabilities (By Formula)

n = 10, p=0.2, find the P(x < 3)

TABLE III

Binomial Probability Distribution

		p										
n	x	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
2	0	0.9801	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500	0.2025
	1	0.0198	0.0950	0.1800	0.2550	0.3200	0.3750	0.4200	0.4550	0.4800	0.4950	0.5000	0.4950
	2	0.0001	0.0025	0.0100	0.0225	0.0400	0.0625	0.0900	0.1225	0.1600	0.2025	0.2500	0.3025
3	0	0.9703	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250	0.0911
	1	0.0294	0.1354	0.2430	0.3251	0.3840	0.4219	0.4410	0.4436	0.4320	0.4084	0.3750	0.3341
	2	0.0003	0.0071	0.0270	0.0574	0.0960	0.1406	0.1890	0.2389	0.2880	0.3341	0.3750	0.4084
	3	0.0000+	0.0001	0.0010	0.0034	0.0080	0.0156	0.0270	0.0429	0.0640	0.0911	0.1250	0.1664
4	0	0.9606	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625	0.0410
	1	0.0388	0.1715	0.2916	0.3685	0.4096	0.4219	0.4116	0.3845	0.3456	0.2995	0.2500	0.2005
	2	0.0006	0.0135	0.0486	0.0975	0.1536	0.2109	0.2646	0.3105	0.3456	0.3675	0.3750	0.3675
	3	0.0000+	0.0005	0.0036	0.0115	0.0256	0.0469	0.0756	0.1115	0.1536	0.2005	0.2500	0.2995

5. Finding Probabilities (By Table)

n = 4, p=0.3, find the P(x = 1)

6. Finding Probabilities (By Table)

n = 4, p=0.45, find the P(x ≤ 2)

TABLE IV

Cumulative Binomial Probability Distribution

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		p												
n	x	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.6
2	0	0.9801	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500	0.2025	0.1
	1	0.9999	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500	0.6975	0.6
	2	1	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0.9703	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250	0.0911	0.0
	1	0.9997	0.9928	0.9720	0.9393	0.8960	0.8438	0.7840	0.7183	0.6480	0.5748	0.5000	0.4253	0.3
	2	1.0000−	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750	0.8336	0.7
	3	1	1	1	1	1	1	1	1	1	1	1	1	1
4	0	0.9606	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625	0.0410	0.0
	1	0.9994	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125	0.2415	0.1
	2	1.0000−	0.9995	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875	0.6090	0.5
	3	1.0000−	1.0000−	0.9999	0.9995	0.9984	0.9961	0.9919	0.9850	0.9744	0.9590	0.9375	0.9085	0.8
	4	1	1	1	1	1	1	1	1	1	1	1	1	1

7. Finding Probabilities (By Table)

n = 4, p=0.45, find the P(x ≤ 2)

TI-83/84:
binompdf (2nd button – vars button) : of this form: binompdf(n,p,x)
where n=# of trials, p=probability of success of one of the trials, and x is the number of successes you want to find)

binomcdf (2nd button – vars button) : of this form: binomcdf(n,p,x)
*same notation as above
this adds all the probabilities from 0 to x (cumulative)

Finding Probabilities

1. Identify number of trials (n)
2. Identify probability of single trial (p). This is often given as a percentage
3. List the x values the problem entails and rewrite with inequality/equality symbols
4. Use the table on the following slide to determine how to enter the function

Forms

Form	TI-83/84 Function
P(x = a)	binompdf(n,p,a)
P(x ≤ a)	binomcdf(n,p,a)
P(x ≥ a)	1-binomcdf(n,p,a-1)
P(a ≤ x ≤ b)	binomcdf(n,p,b)-binomcdf(n,p,a-1)

8. Examples

Given a binomial distribution with n = 30 and p=0.25, find the following:

a) P(exactly 8)
b) P(less than 17)
c) P(at most 12)
d) P(more than 20
e) P(at least 25)
f) P(between 10 and 20, inclusive)

BINOMIAL DISTRIBUTIONS

$$\mu_X = n \cdot p$$

$$\sigma_X = \sqrt{np(1-p)}$$

$$\sigma_X^2 = np(1-p)$$

9. Find the mean, standard deviation and variance

30% of Cowley Students believe in ghosts. Compute the mean, standard deviation, and variance of the random variable X, the number of Cowley students that believe in ghosts based on a random sample of 100 students.