Conditional Probability and the General Multiplication Rule

Definition

Conditional Probability = The notation P(F|E) is read "the probability of event F given event E". It is the probability that the event F occurs, given that the event E has occured

Conditional Probability Rule

$$P(F \mid E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{N(E \text{ and } F)}{N(E)}$$

The probability of event F occurring, given the occurrence of event E, is found by dividing the probability of E and F by the probability of E, or by dividing the number of outcomes in E and F by the number of outcomes in E.

1. Suppose that E and F are two events and that P(E and F) = 0.5 and P(E) = 0.7. What is P(F|E)?
(Similar to p.292 #3-4)

2. Suppose that E and F are two events and that N(E and F) = 300 and N(E) = 800. What is P(F|E)?
(Similar to p.292 #5-6)

3. Suppose that E and F are two events and that P(E) = 0.7 and that P(F|E) = 0.3. What is P(E and F)?
(Similar to p.292 #7-8)

4. Given the general population, 12% are old. In addition 8% are old and forgetful. What is the probability that a randomly selected person will be forgetful given they are old?
(Similar to p.292 #13-16)

5. For 18-20 year olds, 27% believe in aliens and ghosts. Given that 35% of 18-20 years believe in ghosts given they believe in aliens, what probability believe in aliens?
(Similar to p.292 #13-16)

6. Given the following table
(Similar to p.293 #17-20)

	Male	Female	Total
Brunette	25	10	35
Red	2	7	9
Blonde	5	15	20
Total	32	32	64

a) What is the probability that a randomly selected person is female given they have red hair?
b) What is the probability that a randomly selected person has blonde hair given they are male?

General Multiplication Rule
(Dependent Events)

$$P(E \quad and \quad F) = P(E) \cdot P(F \mid E)$$

7. Find the probability

Suppose that three cards are randomly selected from a standard deck of cards:
a) What is the probability of getting three queens with replacement?
b) What is the probability of getting three queens with no replacement?

8. Find the probability

There are ten men and thirteen women in a stats class. Two are to selected to attend a statistic conference. What is the probability that if two are selected at random, that they are both women?

9. Find the probability

Given a standard deck of cards, what is the probability of getting 3 of a kind (3 two's, 3 queen's etc.)? An example hand: Q-Q-Q-2-5 (we cannot have 4 queens since that would be 4 of a kind). Note: (assuming you are dealt the first five cards and the first 3 cards are 3 of the same)

10. Find the probability

In a box, we have 10 black markers, 5 blue markers, 3 red markers and 7 green markers. We are choosing two markers at random:

a) What is the probability they are both green?
b) What is the probability the first is green and the second one is black?
c) What is the probability the first is black and the second one is green?
d) What is the probability the one is black and one is green?

Note

If small random samples are taken from large populations without replacement, it is reasonable to assume independence of the events. As a rule of thumb, if the sample size is less than 5% of the population size, we treat the events as independent.

Definition

Two events E and F are independent if $P(E|F) = P(E)$ or, equivalently, if $P(F|E) = P(F)$