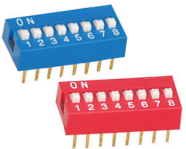# Counting Techniques

## Multiplication Rule of Counting

If a task consists of a sequence of choices in which there are p selections for the first choice, q selections for the second choice, r selections for the third choice, and so on, then the number of possibilities is:

$$p \cdot q \cdot r \cdot \ldots$$

1. A man has two pairs of pants and six shirts, how many different combinations does he have?

2. A red and blue electronics device both have 8 on-off switches, how many different combinations can be set?

### 26 Characters

The full alphabet, either upper or lower case (not both in this case).

| Upper Case Alpha | ABCDEFGHIJKLMNOPQRSTUVWXYZ | |
| Lower Case Alpha | abcdefghijklmnopqrstuvwxyz | |
| **Password** | | |
| Length | Combinations | Class A |
| 2 | 676 | Instant |
| 3 | 17,576 | < 2 Secs |
| 4 | 456,976 | 46 Secs |
| 5 | 11.8 Million | 20 Mins |
| 6 | 308.9 Million | 8½ Hours |
| 7 | 8 Billion | 9 Days |
| 8 | 200 Billion | 242 Days |
| 9 | 5.4 Trillion | 17 Years |
| 10 | 141 Trillion | 447 Years |
| 12 | 95 Quadrillion | 302,603 Years |
| 15 | 1.6 Sextillion | 53 Trillion years |
| 20 | 19.9 Octillion | 63 Quadrillion years |

3. Given a password that consists of the alphabet (A-Z) and case doesn't matter:

a) How many passwords are possible given the password is 5 letters, repetition allowed?

b) How many passwords are possible given the password is 4 letters, repetition not allowed?

4. A social security number is of the form:
xxx-xx-xxxx
where x is a digit from 0 to 9.
Assuming all possibilities are allowed, how many social security numbers are possible?

## Factorial Rule

A collection of n different items can be arranged in order n! different ways

$$n! = 1 \cdot 2 \cdot 3 \cdots (n-1)(n)$$

5. Find 5!

6. How many ways can six students be lined up?

## Permutations Rule
### (Items are all different)

- "Out of n pick r"
- ABC and CBA are different
- Order Matters
- Number of sequences (permutations) of r times selected from n available items is:

$$_nP_r = \frac{n!}{(n-r)!}$$

7. Find $_{10}P_2$

8. Out of a class of 15 students, 3 students are to be picked for president, vice-president, and treasurer of the Statistics Lovers Club, how many possibilities are there?

9. Out of 30 songs, 4 are to be picked for the start of a dance (songs are not to be repeated), how many possibilities are there?

10. A director wants to pick a play for the fall and spring at the college. She has 12 to pick from, how many possibilities are there?

### Combinations Rule

- "Out of n pick r"
- ABC and CBA are same
- Order Doesn't Matter
- Number of combinations of r items selected from n different items is:

$$_nC_r = \frac{n!}{(n-r)!r!}$$

11. Find $_8C_3$

12. Out of a class of 20 students, 4 students are to be picked for representatives for the Statistics Lovers Club, how many possibilities are there?

13. If a child can open three out of their ten presents early on Christmas Eve, how many possibilities are there?

14. If a class has 10 women and 12 men. A committee of 4 women and 3 men are to be chose. How many different possibilities are there?

## Permutations Rule (Some Items are Alike)

- n items, with $n_1$ alike, $n_2$ alike,....,$n_k$ alike

Where

$n = n_1 + n_2 + ... + n_k$

$$\frac{n!}{n_1!n_2!\cdots n_k!}$$

15. How many ways can the letters in MISSISSIPPI be arranged?

## Note

In finding probabilities, the denominator consists of using either permutations or combinations to find the total possibilities.

TI-83 Instructions
nPr -> under "math"-"prb"-"nPr"
nCr -> under "math"-"prb"-"nCr"
! -> under "math"-"prb"-"!"