Distribution of the Sample Mean (Central Limit Theorem)

Given:

$x = random \quad \text{var}\, iable$

$\mu = mean$

$\sigma = std.dev.$

Conclusions:

1. Distribution of sample means will approach a normal distribution.
2. Mean of the sample means will approach the population mean
3. Std. dev. Of sample means will approach population std dev divided by square root of sample size.

Practical Rules

1. For samples of size n larger than 30, distribution of the sample means can be approximated by a normal distribution if the original data is fairly close to normal.
2. If original population is normally distributed, then any sample size will work (within reason).

Formulas

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

1. Determine $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ from the given parameters of the population and the sample size.

μ = 40, σ = 5, n = 20

Central Limit Theorem

Regardless of the shape of the underlying population, the sampling distribution of $\bar{x}$ becomes approximately normal as the sample size, n, increases

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$$normalcdf\,(lowerbound, upperbound, \mu, \frac{\sigma}{\sqrt{n}})$$

2. Test Scores

Test grades are normally distributed with a mean of 70 and a standard deviation of 2:

a) What is the probability of picking a test grade out and getting one less than 69
b) What is the probability that a random sample of 8 test grades have a mean less than 69
c) What is the probability that a random sample of 15 test grades have a mean less than 69

3. Test Scores

Test grades are normally distributed with a mean of 70 and a standard deviation of 2:

a) What is the probability of picking a test grade out and getting one more than 69
b) What is the probability that a random sample of 10 test grades have a mean more than 69
c) What is the probability that a random sample of 20 test grades have a mean more than 69

4. Test Scores

Test grades are normally distributed with a mean of 70 and a standard deviation of 2:

a) What is the probability of picking a test grade out and getting one between 60 and 71
b) What is the probability that a random sample of 10 test grades have a mean between 60 and 71
c) What is the probability that a random sample of 20 test grades have a mean between 60 and 71

5. Test Scores

Given 30 test scores with a sample mean of 80 and a standard deviation of 5, the value that separates the top 10% is _____.