Inference about Two Population Standard Deviations

Testing Hypothesis Regarding Two Population Standard Deviations

- The samples are obtained using simple random sampling or through a randomized experiment
- The samples are independent
- The populations from which the samples are drawn are normally distributed

Fisher's F-distribution

If $\sigma_1^2 = \sigma_2^2$ and s_1^2 *and* s_2^2 are sample variances from independent simple random samples of size n_1 and n_2, respectively, drawn from normal populations, then

$$F = \frac{s_1^2}{s_2^2}$$

Follows the F-distribution with $n_1 - 1$ degrees of freedom in the numerator and $n_2 - 1$ degrees of freedom in the denominator

Classical Approach

1. Write down a shortened version of claim
2. Come up with null and alternate hypothesis (H_o always has the equals part on it)
3. See if claim matches H_o or H_1
4. Draw the picture and split α into tail(s)

   $H_1: \sigma_1 \neq \sigma_2$ Two Tail

   $H_1: \sigma_1 < \sigma_2$ Left Tail

   $H_1: \sigma_1 > \sigma_2$ Right Tail

Classical Approach)

5. Find critical values (F-Distribution table)

   Numerator DF = $n_1 - 1$, Denominator DF = $n_2 - 1$

   Left Tail

   Left CV = $\frac{1}{F_{\alpha, n_2-1, n_1-1}}$

   Right Tail

   Right CV = F_{α, n_1-1, n_2-1}

   Two Tails

   Left CV = $\frac{1}{F_{\alpha/2, n_2-1, n_1-1}}$

   Right CV = $F_{\alpha/2, n_1-1, n_2-1}$

Classical Approach

6. Find test statistic

   TI-83/84: 2-SampFTest

   By Hand: $F_o = \frac{s_1^2}{s_2^2}$

7. If test statistic falls in tail, Reject H_o. If test statistic falls in main body, Accept H_o. Determine the claim based on step 3

P-Value Approach (TI-83/84)

1. Write down a shortened version of claim
2. Come up with null and alternate hypothesis (H_o always has the equals part on it)
3. See if claim matches H_o or H_1
4. Find p-value (2-SampFTest)
5. If p-value is less than α, Reject H_o. If p-value is greater than α, Accept H_o. Determine the claim based on step 3

1. Critical Values

Find the critical value for a right tailed test with $\alpha = 0.05$, degrees of freedom in the numerator = 20, and degrees of freedom in the denominator = 25

2. Critical Values

Find the critical value for a left tailed test with $\alpha = 0.10$, degrees of freedom in the numerator = 6, and degrees of freedom in the denominator = 10

3. Critical Values

Find the critical values for a two-tailed test with $\alpha = 0.05$, degrees of freedom in the numerator = 5, and degrees of freedom in the denominator = 8

4. Claim

Test the hypothesis that $\sigma_1 \neq \sigma_2$ at the $\alpha = 0.05$ level of significance for the given sample data

	Population 1	Population 2
n	8	6
s	3.3	3.6

5. Claim

Test the hypothesis that $\sigma_1 > \sigma_2$ at the $\alpha = 0.05$ level of significance for the given sample data

	Population 1	Population 2
n	22	12
s	4.2	4

6. Claim

Test the hypothesis that $\sigma_1 < \sigma_2$ at the $\alpha = 0.01$ level of significance for the given sample data

	Population 1	Population 2
n	11	8
s	5	7.2