Trigonometry
Chapter 1/2 Test Review Key

Instructions: All answers should be in exact form, no decimals.

1. Find the center and radius of the following:

$x^2 + y^2 - 10x + 8y + 32 = 0$

$x^2 + y^2 - 10x + 8y + 32 = 0$

$x^2 - 10x + y^2 + 8y = -32$

$x^2 - 10x + 25 + y^2 + 8y + 16 = -32 + 25 + 16$

$(x-5)^2 + (y+4)^2 = 9$

so

$Center = (5,-4)$

$Radius = 3$

x's	y's
$\left(-10 \cdot \frac{1}{2}\right)^2$	$\left(8 \cdot \frac{1}{2}\right)^2$
$(-5)^2$	$(4)^2$
25	16

$4x^2 + 4y^2 - 4x + 24y + 21 = 0$

$4x^2 + 4y^2 - 4x + 24y + 21 = 0$

$\frac{4x^2}{4} + \frac{4y^2}{4} - \frac{4x}{4} + \frac{24y}{4} + \frac{21}{4} = \frac{0}{4}$

$x^2 + y^2 - x + 6y + \frac{21}{4} = 0$

$x^2 - x + y^2 + 6y = \frac{-21}{4}$

$x^2 - x + \frac{1}{4} + y^2 + 6y + 9 = \frac{-21}{4} + \frac{1}{4} + 9$

$\left(x - \frac{1}{2}\right)^2 + (y+3)^2 = 4$

so

$Center = \left(\frac{1}{2}, -3\right)$

$Radius = 2$

x's	y's
$\left(-1 \cdot \frac{1}{2}\right)^2$	$\left(6 \cdot \frac{1}{2}\right)^2$
$\left(\frac{-1}{2}\right)^2$	$(3)^2$
$\frac{1}{4}$	9

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2. Convert the following (no decimals):

35° to radians $\frac{35^\circ}{1}\cdot\frac{\pi}{180^\circ}=\frac{35\pi}{180}=\frac{7\pi}{36}$	80° to radians $\frac{80^\circ}{1}\cdot\frac{\pi}{180^\circ}=\frac{80\pi}{180}=\frac{8\pi}{18}=\frac{4\pi}{9}$
$\frac{4\pi}{9}$ to degrees $\frac{\frac{4\pi}{9}}{1}\cdot\frac{180^\circ}{\pi}=\frac{4\pi}{9}\cdot\frac{180^\circ}{\pi}=\frac{4}{1}\cdot\frac{20^\circ}{1}=80^\circ$	$\frac{2\pi}{5}$ to degrees $\frac{\frac{2\pi}{5}}{1}\cdot\frac{180^\circ}{\pi}=\frac{2\pi}{5}\cdot\frac{180^\circ}{\pi}=\frac{2}{1}\cdot\frac{36^\circ}{1}=72^\circ$

3. Find exact values of trig functions based on the unit circle:

Find the exact value of: $\cot\frac{-\pi}{3}-\sec\frac{\pi}{6}$

$$\cot\frac{-\pi}{3}-\sec\frac{\pi}{6}$$
$$=-\cot\frac{\pi}{3}-\sec\frac{\pi}{6}$$
$$=-\left(\frac{1/2}{\sqrt{3}/2}\right)-\left(\frac{1}{\sqrt{3}/2}\right)$$
$$=-\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}$$
$$=\frac{-3}{\sqrt{3}}$$
$$=\frac{-3\sqrt{3}}{3}$$
$$=-\sqrt{3}$$

Find the exact value of:
$7\csc\frac{3\pi}{4}-\cot\left(\frac{-2\pi}{3}\right)$

$$7\csc\frac{3\pi}{4}-\cot\left(\frac{-2\pi}{3}\right)$$
$$=7\csc\frac{3\pi}{4}+\cot\left(\frac{2\pi}{3}\right)$$
$$=7\left(\frac{1}{\sqrt{2}/2}\right)+\left(\frac{-1/2}{\sqrt{3}/2}\right)$$
$$=7\left(\frac{2}{\sqrt{2}}\right)-\frac{1}{\sqrt{3}}$$
$$=\frac{14}{\sqrt{2}}-\frac{1}{\sqrt{3}}$$
$$=\frac{14\sqrt{2}}{2}-\frac{\sqrt{3}}{3}$$
$$=\frac{7\sqrt{2}}{1}-\frac{\sqrt{3}}{3}$$
$$=\frac{21\sqrt{2}}{3}-\frac{\sqrt{3}}{3}$$
$$=\frac{21\sqrt{2}-\sqrt{3}}{3}$$

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Find the exact value of: $\sin\frac{\pi}{3}-\cos\frac{\pi}{2}$

$$\sin\frac{\pi}{3}-\cos\frac{\pi}{2}$$
$$=\left(\frac{\sqrt{3}}{2}\right)-(0)$$
$$=\frac{\sqrt{3}}{2}$$

Find the exact value of:
$5\sec\frac{9\pi}{4}-\csc\left(\frac{-2\pi}{3}\right)$

$$5\sec\frac{9\pi}{4}-\csc\left(\frac{-2\pi}{3}\right)$$
$$=5\sec\left(\frac{9\pi}{4}-2\pi\right)+\csc\left(\frac{2\pi}{3}\right)$$
$$=5\sec\left(\frac{\pi}{4}\right)+\csc\left(\frac{2\pi}{3}\right)$$
$$=5\left(\frac{1}{\sqrt{2}/2}\right)+\left(\frac{1}{\sqrt{3}/2}\right)$$
$$=5\left(\frac{2}{\sqrt{2}}\right)+\frac{2}{\sqrt{3}}$$
$$=\frac{10}{\sqrt{2}}+\frac{2}{\sqrt{3}}$$
$$=\frac{10\sqrt{2}}{2}+\frac{2\sqrt{3}}{3}$$
$$=\frac{5\sqrt{2}}{1}+\frac{2\sqrt{3}}{3}$$
$$=\frac{15\sqrt{2}}{3}+\frac{2\sqrt{3}}{3}$$
$$=\frac{15\sqrt{2}+2\sqrt{3}}{3}$$

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4. Find the exact value of the trigonometric functions given various criteria:

Find the exact value of each of the remaining trigonometric functions: $\csc\theta = -7, \tan\theta < 0$	Find the exact value of each of the remaining trigonometric functions: $\cos\theta = \frac{3}{7}, \sin\theta < 0$
It is in QIV	It is in QIV
$\csc\theta = \frac{-7}{1} = \frac{7}{-1} = \frac{r}{y}$	$\cos\theta = \frac{3}{7} = \frac{x}{r}$
$so \quad r = 7, y = -1$	$so \quad x = 3, r = 7$
$x^2 + y^2 = r^2$	$x^2 + y^2 = r^2$
$x^2 + (-1)^2 = 7^2$	$(3)^2 + y^2 = 7^2$
$x^2 + 1 = 49$	$9 + y^2 = 49$
$x^2 = 49 - 1$	$y^2 = 49 - 9$
$x^2 = 48$	$y^2 = 40$
$x = \pm\sqrt{48}$	$y = \pm\sqrt{40}$
$x = \pm\sqrt{(2)(2)(2)(2)(3)}$	$y = \pm\sqrt{(2)(2)(2)(5)}$
$x = \pm 4\sqrt{3}$	$y = \pm 2\sqrt{10}$
$x = 4\sqrt{3}$	$y = -2\sqrt{10}$
Then	Then
$x = 4\sqrt{3}, y = -1, r = 7$	$x = 3, y = -2\sqrt{10}, r = 7$
$\cos\theta = \frac{x}{r} = \frac{4\sqrt{3}}{7}$	$\cos\theta = \frac{x}{r} = \frac{3}{7}$
$\sin\theta = \frac{y}{r} = \frac{-1}{7}$	$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{10}}{7}$
$\tan\theta = \frac{y}{x} = \frac{-1}{4\sqrt{3}} = \frac{-\sqrt{3}}{12}$	$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{10}}{3}$
$\sec\theta = \frac{7}{4\sqrt{3}} = \frac{7\sqrt{3}}{12}$	$\sec\theta = \frac{7}{3}$
$\csc\theta = \frac{7}{-1} = -7$	$\csc\theta = \frac{7}{-2\sqrt{10}} = \frac{7\sqrt{10}}{-20}$
$\cot\theta = \frac{x}{y} = \frac{4\sqrt{3}}{-1} = -4\sqrt{3}$	$\cot\theta = \frac{x}{y} = \frac{3}{-2\sqrt{10}} = \frac{3\sqrt{10}}{-20}$

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Find the exact value of each of the remaining trigonometric functions:

$$\sec\theta = 5, \sin\theta > 0$$

It in QI

$\sec\theta = \frac{5}{1} = \frac{r}{x}$

$so \quad r = 5, x = 1$

$x^2 + y^2 = r^2$

$(1)^2 + y^2 = 5^2$

$1 + y^2 = 25$

$y^2 = 25 - 1$

$y^2 = 24$

$y = \pm\sqrt{24}$

$y = \pm\sqrt{(2)(2)(2)(3)}$

$y = \pm 2\sqrt{6}$

$y = 2\sqrt{6}$

Then

$x = 1, y = 2\sqrt{6}, r = 5$

$\cos\theta = \frac{x}{r} = \frac{1}{5}$

$\sin\theta = \frac{y}{r} = \frac{2\sqrt{6}}{5}$

$\tan\theta = \frac{y}{x} = \frac{2\sqrt{6}}{1} = 2\sqrt{6}$

$\sec\theta = \frac{5}{1} = 5$

$\csc\theta = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$

$\cot\theta = \frac{x}{y} = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$

Find the exact value of each of the remaining trigonometric functions:

$$\sin\theta = \frac{2}{3}, \tan\theta > 0$$

It in QI

$\sin\theta = \frac{2}{3} = \frac{y}{r}$

$so \quad y = 2, r = 3$

$x^2 + y^2 = r^2$

$x^2 + (2)^2 = 3^2$

$x^2 + 4 = 9$

$x^2 = 9 - 4$

$x^2 = 5$

$x = \pm\sqrt{5}$

$x = \sqrt{5}$

Then

$x = \sqrt{5}, y = 2, r = 3$

$\cos\theta = \frac{x}{r} = \frac{\sqrt{5}}{3}$

$\sin\theta = \frac{y}{r} = \frac{2}{3}$

$\tan\theta = \frac{y}{x} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\sec\theta = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$\csc\theta = \frac{3}{2}$

$\cot\theta = \frac{x}{y} = \frac{\sqrt{5}}{2}$

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5. Find the amplitude and/or period:

Find the amplitude and period of $y = 4\cos(2x)$ $\lvert a\rvert = \lvert 4\rvert = 4$ $period = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$	Find the amplitude and period of $y = -2\sin\left(\frac{\pi}{2}x\right)$ $\lvert a\rvert = \lvert -2\rvert = 2$ $period = \frac{2\pi}{b} = \frac{2\pi}{\pi/2} = 2\pi \cdot \frac{2}{\pi} = 4$
Find the period of: $y = 5\csc(\pi x)$ $period = \frac{2\pi}{b} = \frac{2\pi}{\pi} = 2$	Find the period of: $y = 2\tan(4x)$ $period = \frac{\pi}{b} = \frac{\pi}{4}$

6. Find the period and any phase shifts (vertical or horizontal shifts) of:

$y = -8 + 3\tan(4\theta - 9)$ $y = -8 + 3\tan\left[4\left(\theta - \frac{9}{4}\right)\right]$ *down* 8 *right* $\frac{9}{4}$ $period = \frac{\pi}{b} = \frac{\pi}{4}$	$y = -2 + 4\cot(7\theta + 2)$ $y = -2 + 4\cot\left[7\left(\theta + \frac{2}{7}\right)\right]$ *down* 2 *left* $\frac{2}{7}$ $period = \frac{\pi}{b} = \frac{\pi}{7}$

7. Graph:
$y = 4\sin(2x)$

PERIOD $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

	X	$y = 4\sin(2x)$
Z	0	$4\sin(2\cdot 0) = 4\sin 0 = 4(0) = 0$
$\frac{1}{4}$P	$\frac{\pi}{4}$	$4\sin(2\cdot\frac{\pi}{4}) = 4\sin\frac{\pi}{2} = 4(1) = 4$
$\frac{1}{2}$P	$\frac{\pi}{2}$	$4\sin(2\cdot\frac{\pi}{2}) = 4\sin\pi = 4(0) = 0$
$\frac{3}{4}$P	$\frac{3\pi}{4}$	$4\sin(2\cdot\frac{3\pi}{4}) = 4\sin\frac{3\pi}{2} = 4(-1) = -4$
P	π	$4\sin(2\cdot\pi) = 4\sin 2\pi = 4(0) = 0$

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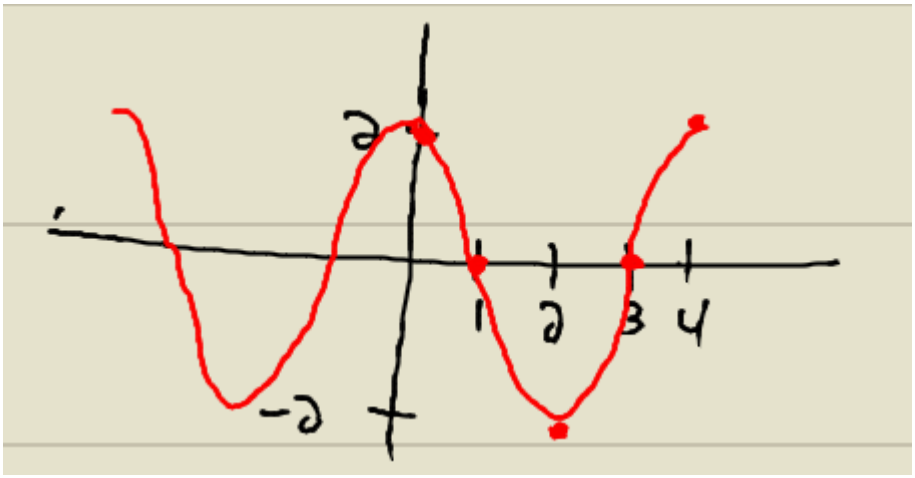
$y = 2\cos\left(\frac{\pi}{2}x\right)$

$\text{PERIOD} = \frac{2\pi}{b} = \frac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \frac{2}{\pi} = 4$

	X	$y = 2\cos\left(\frac{\pi}{2}x\right)$
Z	0	$2\cos\left(\frac{\pi}{2}\cdot 0\right) = 2\cos 0 = 2(1) = 2$
$\frac{1}{4}$P	1	$2\cos\left(\frac{\pi}{2}\cdot 1\right) = 2\cos\frac{\pi}{2} = 2(0) = 0$
$\frac{1}{2}$P	2	$2\cos\left(\frac{\pi}{2}\cdot 2\right) = 2\cos\pi = 2(-1) = -2$
$\frac{3}{4}$P	3	$2\cos\left(\frac{\pi}{2}\cdot 3\right) = 2\cos\frac{3\pi}{2} = 2(0) = 0$
P	4	$2\cos\left(\frac{\pi}{2}\cdot 4\right) = 2\cos(2\pi) = 2(1) = 2$



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8. Given a point not on the unit circle, find the value of the trigonometric functions:

The point given is on the terminal side of an angle in standard position. Find the exact values of the trigonometric functions: (-2, -11)

$x = -2, y = -11$

$r^2 = x^2 + y^2$

$r^2 = (-2)^2 + (-11)^2$

$r^2 = 4 + 121$

$r^2 = 125$

$r = \sqrt{125}$

$r = \sqrt{(5)(5)(5)}$

$r = 5\sqrt{5}$

then

$\cos\theta = \frac{x}{r} = \frac{-2}{5\sqrt{5}} = \frac{-2\sqrt{5}}{25}$

$\sin\theta = \frac{y}{r} = \frac{-11}{5\sqrt{5}} = \frac{-11\sqrt{5}}{25}$

$\tan\theta = \frac{y}{x} = \frac{-11}{-2} = \frac{11}{2}$

$\sec\theta = \frac{5\sqrt{5}}{-2}$

$\csc\theta = \frac{5\sqrt{5}}{-11}$

$\cot\theta = \frac{2}{11}$

The point given is on the terminal side of an angle in standard position. Find the exact values of the trigonometric functions: (-3, 10)

$x = -3, y = 10$

$r^2 = x^2 + y^2$

$r^2 = (-3)^2 + (10)^2$

$r^2 = 9 + 100$

$r^2 = 109$

$r = \sqrt{109}$

then

$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{109}} = \frac{-3\sqrt{109}}{109}$

$\sin\theta = \frac{y}{r} = \frac{10}{\sqrt{109}} = \frac{10\sqrt{109}}{109}$

$\tan\theta = \frac{y}{x} = \frac{10}{-3}$

$\sec\theta = \frac{\sqrt{109}}{-3}$

$\csc\theta = \frac{\sqrt{109}}{10}$

$\cot\theta = \frac{-3}{10}$

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9. Use even/odd properties and/or the periodic nature of trigonometric functions to find the exact value of an expression:

Find the exact value of $\tan 420^\circ$. $\tan 420^\circ$ $= \tan(420 - 360)^\circ$ $= \tan 60^\circ$ $= \dfrac{\sqrt{3}/2}{1/2}$ $= \dfrac{\sqrt{3}}{1}$ $= \sqrt{3}$	Find the exact value of $\csc 1860^\circ$. $\csc 1860^\circ$ $= \csc(1860 - 1800)^\circ$ $= \csc 60^\circ$ $= \dfrac{1}{\sqrt{3}/2}$ $= \dfrac{2}{\sqrt{3}}$ $= \dfrac{2\sqrt{3}}{3}$
Find the exact value of $\cos^2 60^\circ + \sin^2(420^\circ)$ $\cos^2 60^\circ + \sin^2(420^\circ)$ $= \cos^2 60^\circ + \sin^2(420^\circ - 360^\circ)$ $= \cos^2 60^\circ + \sin^2(60^\circ)$ $= 1$	Find the exact value of $\tan 30^\circ \cdot \cot(-150^\circ)$ $\tan 30^\circ \cdot \cot(-150^\circ)$ $= \tan 30^\circ \cdot \cot(-150^\circ + 180^\circ)$ $= \tan 30^\circ \cdot \cot(30^\circ)$ $= \tan 30^\circ \cdot \dfrac{1}{\tan 30^\circ}$ $= 1$