The Complex Plane; De Moivre's Theorem

Polar Form

$$z = x + yi = r(\cos\theta + i\sin\theta)$$
$$r \geq 0$$
$$0 \leq \theta < 2\pi$$

1. Plot each complex number in the complex plane and write it in polar form. Express the argument in degrees
(Similar to p.334 #11-22)

$$-1 - i$$

2. Plot each complex number in the complex plane and write it in polar form. Express the argument in degrees
(Similar to p.334 #11-22)

$$5\sqrt{3} - 5i$$

3. Plot each complex number in the complex plane and write it in polar form. Express the argument in degrees
(Similar to p.334 #11-22)

$$\sqrt{7} - i$$

4. Write each complex number in rectangular form
(Similar to p.334 #23-32)

$$5(\cos\pi + i\sin\pi)$$

5. Write each complex number in rectangular form
(Similar to p.334 #23-32)

$$4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$

6. Write each complex number in rectangular form
(Similar to p.334 #23-32)

$$7\left(\cos 22^\circ + i\sin 22^\circ\right)$$

Multiplication and Division

$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

then

$$z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\right]$$

7. Find zw and z/w. Leave your answers in polar form
(Similar to p.334 #33-40)

$$z = \cos 200^\circ + i\sin 200^\circ$$

$$w = \cos 120^\circ + i\sin 120^\circ$$

8. Find zw and z/w. Leave your answers in polar form
(Similar to p.334 #33-40)

$$z = 8\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

$$w = 4\left(\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}\right)$$

9. Find zw and z/w. Leave your answers in polar form
(Similar to p.334 #33-40)

$$z = 3-3i$$

$$w = -\sqrt{3}+i$$

De Moivre's Theorem

$z = r(\cos\theta + i\sin\theta)$

then

$z^n = r^n\left[\cos(n\theta) + i\sin(n\theta)\right]$

$(n \geq 1)$

10. Write each expression in the standard form a + bi
(Similar to p.334 #41-52)

$$\left[2\left(\cos 105^\circ + i\sin 105^\circ\right)\right]^4$$

11. Write each expression in the standard form a + bi
(Similar to p.334 #41-52)

$$\left[\sqrt{2}\left(\cos\frac{3\pi}{20} + i\sin\frac{3\pi}{20}\right)\right]^{10}$$

Let w = r(cos θ_0 + i sin θ_0 be a complex number, and let n $\geq$ 2 be an integer. There are n distinct complex nth roots given by:

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right)\right]$$

where $k = 0,1,2,\ldots,n-1$

12. Find all the complex roots. Leave your answers in polar form with the argument in degrees
(Similar to p.335 #53-60)

The complex fourth roots of $4 - 4i$

13. Solve the following equation. Leave your answers in polar form with the argument in degrees
(Similar to p.335 #53-60)

$$x^5 = -27$$